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SOLUTIONS OF EXERCISES.

1

A horizontal wind blows against a hemispherical dome of radius R^f. The pressure exerted by the wind on a plane surface normal to its direction is P pounds to the square foot; on a surface oblique to its direction the pressure exerted is normal, but is reduced in the ratio (Poncelet, *Mécanique Industrielle*, 403)

$$1 : 1 + \frac{1}{2}\tan^2 i,$$

where i is the angle of incidence. It is required to find the magnitudes and the points of application of the horizontal and vertical components of the resultant wind-pressure. [*W. M. Thornton.*]

SOLUTION.

Take the axis of x in the direction of the wind, and the plane of xy for the horizontal plane. The element of the surface is

$$dA = \sqrt{1 + p^2 + q^2} \, . \, dx\, dy;$$

and since the equation of the surface is $x^2 + y^2 + z^2 = R^2$,

$$p = \frac{dz}{dx} = -\frac{x}{z}, \qquad q = \frac{dz}{dy} = -\frac{y}{z}, \qquad dA = \frac{R}{z} \, . \, dx\, dy.$$

We have also $\cos i = \frac{x}{R}$, and therefore

$$1 + \frac{1}{2}\tan^2 i = \frac{R^2 + x^2}{2x^2}.$$

The normal pressure on the element of surface is

$$\frac{2PRx^2}{z\,(R^2 + x^2)} \, . \, dx\, dy.$$

Put $a^2 = R^2 - x^2$, so that $z = \sqrt{a^2 - y^2}$; and since we have to multiply the normal pressure by $\frac{x}{R}$ to get the horizontal component, the elementary pressure parallel to the axis of x is

$$\frac{2Px^3\,dx}{R^2 + x^2} \cdot \frac{dy}{\sqrt{a^2 - y^2}}.$$

The limits for y are 0 and a, and integrating with respect to y, and denoting the horizontal pressure by H we have

$$H = 2\pi P.\int_0^R \frac{x^3\,dx}{R^2+x^2} = 2\pi P.\left\{\frac{x^2}{2} - \frac{R^2}{2}\log(R^2+x^2)\right\}_0^R$$

or

$$H = \pi P R^2.(1-\log 2). \qquad (1)$$

For the x co-ordinate of the point of application of the horizontal force we have to find the integral

$$\int_0^R \frac{x^4\,dx}{R^2+x^2} = \left(\frac{x^3}{3} - Rx^2 + R^3\arctan\frac{x}{R}\right)_0^R$$

$$= \left(\frac{\pi}{4} - \frac{2}{3}\right).R^3,$$

and hence

$$\bar{x} = \frac{3\pi - 8}{6(1-\log 2)}.R. \qquad (2)$$

For the corresponding z co-ordinate we have

$$\frac{4P.x^3\,dx}{R^2+x^2}\int_0^a dy = \frac{4P.x^3\sqrt{R^2-x^2}.dx}{R^2+x^2}$$

and

$$\int\frac{x^3\sqrt{R^2-x^2}}{R^2+x^2}.dx = R^2.\int\frac{x^3\,dx}{(R^2+x^2)\sqrt{R^2-x^2}} - \int\frac{x^5\,dx}{(R^2+x^2)\sqrt{R^2-x^2}}.$$

But

$$R^2.\int_0^R\frac{x^3\,dx}{(R^2+x^2)\sqrt{R^2-x^2}} = \left[1+\frac{1}{\sqrt{2}}.\log(\sqrt{2}-1)\right].R^3$$

and

$$\int_0^R\frac{x^5\,dx}{(R^2+x^2)\sqrt{R^2-x^2}} = -\left\{\frac{1}{3}+\frac{1}{\sqrt{2}}.\log(\sqrt{2}-1)\right\}R^3,$$

so that we have

$$\bar{z} = \frac{4[4+3\sqrt{2}.\log(\sqrt{2}-1)]}{3\pi(1-\log 2)}.R. \qquad (3)$$

In order to get the vertical pressure on the element we have to multiply by $\frac{z}{R}$, and we have for this elementary pressure

$$\frac{2Px^2}{R^2 + x^2} \cdot dx\, dy.$$

The integration with respect to y gives

$$\frac{4P \cdot x^2\sqrt{R^2 - x^2}}{R^2 + x^2} \cdot dx,$$

and hence the vertical pressure is

$$4P \cdot \int_0^R \frac{x^2\sqrt{R^2 - x^2}}{R^2 + x^2} \cdot dx.$$

If V be the vertical pressure the reduction of this integral gives

$$V = \pi PR^2 \cdot (3 - \sqrt{8}). \tag{4}$$

For the x co-ordinate of V we have to find the integral of

$$\frac{2Px^3}{R^2 + x^2}\, dx\, dy.$$

Integrating with respect to y we have

$$4P \cdot \int_0^R \frac{x^3\sqrt{R^2 - x^2}}{R^2 + x^2} \cdot dx.$$

This integral is the same as that found before, and we have

$$\bar{x} = \frac{4\,[4 + 3\sqrt{2} \cdot \log(\sqrt{2} - 1)]}{3\pi\,(3 - \sqrt{8})} \cdot R. \tag{5}$$

For the z co-ordinate we have the form

$$\frac{2Px^2z}{R^2 + x^2}\, dx\, dy = \frac{2Px^2\, dx}{R^2 + x^2} \cdot \sqrt{a^2 - y^2} \cdot dy.$$

But

$$\int_0^a \sqrt{a^2 - y^2} \cdot dy = \frac{a^2\pi}{4} = (R^2 - x^2) \cdot \frac{\pi}{4};$$

and also

$$\pi P \cdot \int_0^R \frac{x^2(R^2 - x^2)}{R^2 + x^2} \cdot dx = \pi P \left(\frac{5}{3} - \frac{\pi}{2}\right) \cdot R^3.$$

We have therefore

$$z = \frac{10 - 3\pi}{6(3 - \sqrt{8})} \, . \, R \, . \tag{6}$$

Collecting results and expressing the coefficients by common logarithms we have

$$\begin{aligned} H &= [9.98408] \, . \, PR^2, & V &= [9.73160] \, . \, PR^2, \\ \bar{x} &= [9.88867] \, . \, R, & \bar{x} &= [9.80943] \, . \, R, \\ \bar{z} &= [9.55695] \, . \, R, & \bar{z} &= [9.74724] \, . \, R. \end{aligned}$$

If we assume $P = 100$, and $R = 22$ feet, we have

$$\begin{aligned} H &= 46658 \text{ pounds}, & V &= 26088 \text{ pounds}, \\ \bar{x} &= 17.0 \text{ feet}, & \bar{x} &= 14.2 \text{ feet}, \\ \bar{z} &= 7.9 \text{ feet}, & \bar{z} &= 12.3 \text{ feet}. \end{aligned}$$

[*A. Hall.*]

[The formula quoted in the question is empirical and was derived by Duchemin from experimental results given by Vince, (*Philosophical Transactions of the Royal Society of London*, 1778); by Hutton, (*Resistance of the Air to Bodies in Motion*, Tract 36, 1788); and by Thibault, (*Recherches experimentales sur la resistance de l'air*). Hutton represented the results of his own experiments alone quite exactly by the formula

$$(\cos i)^{1.842 \sin i}.$$

This formula represents the whole group of data however less well than Duchemin's.—Ed.]

EXERCISES.

7

In the theory of perturbations, if the differential equations have the form

$$\frac{d^2\xi}{dt^2} + \frac{k^2(1 + m)}{r^3} \, . \, \xi = A,$$

prove that

$$k\sqrt{p_0} \, . \, \xi = \xi_0 + \xi_1 + \xi_2 + \ldots \, ,$$

where

$$\xi_0 = y_0 \int A x_0 \, dt - x_0 \int A y_0 \, dt + c_1 y_0 - c_2 x_0 \, ,$$